

Uses and Misuses of Logic

Introduction

This document contains observations on the uses and misuses of logic, particularly in the sciences. Along the way we'll wander into the murky realms of absolute and proximate truths, deduction and induction and address the question of how we can have confidence in knowledge that is less than perfect.

We will use certain terms as scientists use them. For those not familiar with the language of science, we include here some fundamentals, so we'll all be starting with the same language.

Fact. An isolated piece of information about nature. It can be simply a measurement. Sometimes related facts are called "data".

Hypothesis. A proposition about nature that is testable, but not yet tested to the point of general acceptance.

Law. A statement describing how some phenomenon of nature behaves. Laws are generalizations from data. They express regularities and patterns in the data. A law is usually limited in scope, to describe a particular process of nature.

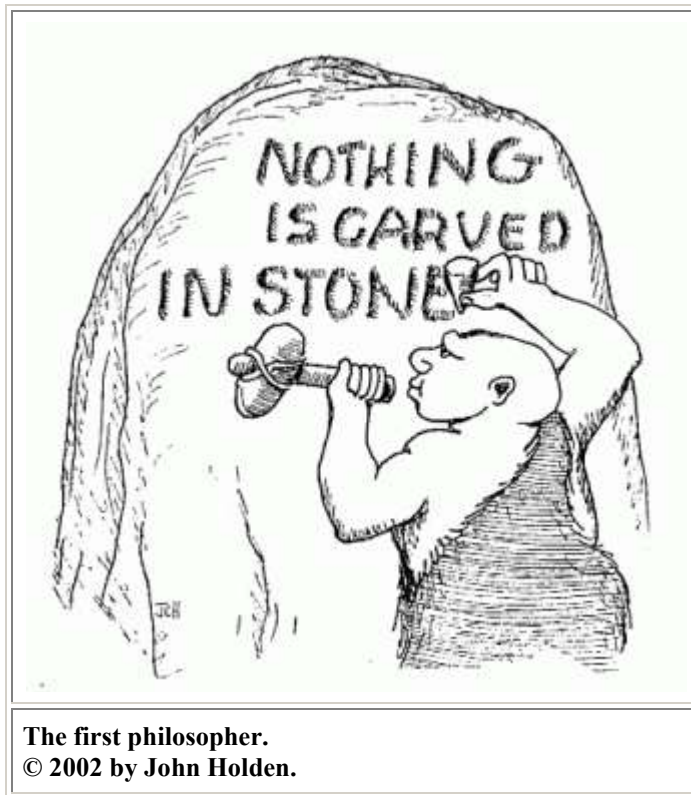
Theory. A model (usually mathematical) that links and unifies a broader range of phenomena, and that links and synthesizes the laws that describe those phenomena. In science we do not grant an idea the status of **theory** until its consequences have been very well tested and are generally accepted as correct by knowledgeable scientists. This meaning is very different from colloquial use of the word.

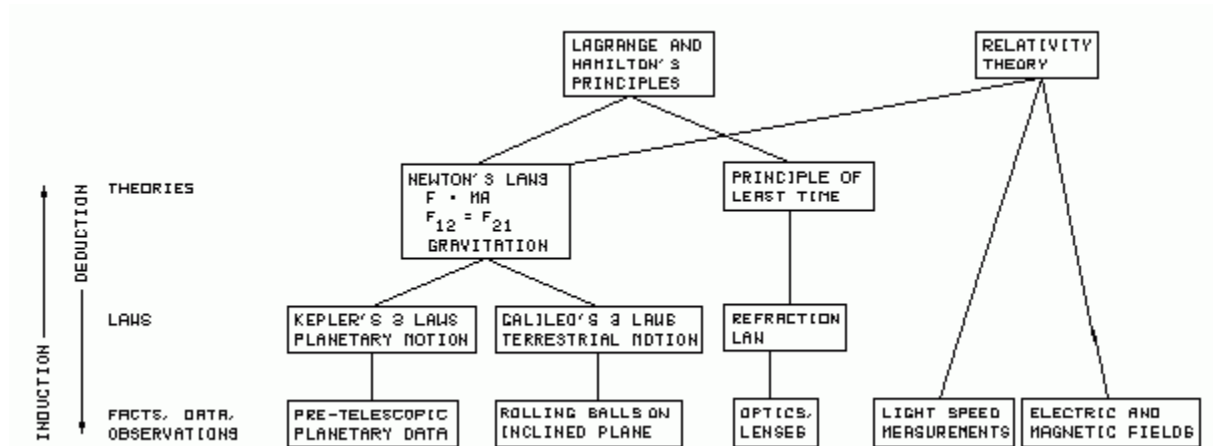
Induction and deduction

Science proceeds from facts to laws to theories by a difficult-to-define process called **induction**. Induction includes pattern-recognition, brainstorming, tinkering, creative guessing and that elusive "insight". It is **not** a process of deductive logic.

Theories and laws are required to be of such form that one can deductively proceed from theories to laws to data. The results of deduction must meet a stringent standard: they must agree with experiment and with observations of nature.

Mathematics is a process of deductive logic. Therefore it is ideally suited to be the language and the deductive link between theories and experimental facts. Because of this, some non-scientists think that mathematics and logic are used to "prove" scientific propositions, to deduce new laws and theories, and to establish laws and theories with mathematical certainty. This is false, as we shall see.





This diagram shows the relations between facts, laws, and theories, and the role of induction and deduction. It will take on more meaning as this essay progresses.

Quotes about logic

Logic is the art of going wrong with confidence.
Joseph Wood Krutch

Logic: an instrument used for bolstering a prejudice.
Elbert Hubbard

It is always better to say right out what you think without trying to prove anything much: for all our proofs are only variations of our opinions, and the contrary-minded listen neither to one nor the other.
Johann Wolfgang von Goethe (1749-1832)

Most of our so-called reasoning consists in finding arguments for going on believing as we already do.
James Harvey Robinson

Logic is neither a science nor an art, but a dodge.
Benjamin Jowett

Logic, like whiskey, loses its beneficial effect when taken in too large quantities.
Lord Dunsany

He was in Logic a great critic,
Profoundly skill'd in Analytic;
He could distinguish, and divide
A hair 'twixt south and south-west side.
Samuel Butler, Hudibras.

We must beware of needless innovations, especially when guided by logic.
Sir Winston Churchill, Reply, House of Commons, Dec. 17, 1942.

...logic, the refuge of fools. The pedant and the priest have always been the most expert of logicians—and the most diligent disseminators of nonsense and worse.
H. L. Mencken. The American Mercury. p. 75.

Formal logic, uses and misuses

Formal logic was invented in Classical Greece and integrated into a 'system' of thought by Aristotle. It was, for him, a tool for finding truth, but it didn't keep him from making the most profound errors of thought. Nearly every argument and conclusion he made about physical science was wrong and misguided. Any tool can be misused, and in these pre-scientific days logic was misused repeatedly.

So what went wrong? Aristotle understood that logic can be used to deduce true consequences from true premises. His error was his failure to realize that we have no absolutely true premises, except ones we *define* to be true (such as $2+2=4$). Aristotle thought that the mind contains (from birth) some innate and absolutely true knowledge that can be used as premises for logical arguments. Medieval scholastics, who brought Aristotelian modes of thought to a height of absurdity, thought that absolutely true premises could be found in revelations from God, as recorded in the Bible.

Another error was to assume that the conclusions from a logical argument represent new truths. In fact, the deduced conclusions are just restatements and repackaging of the content contained in the premises. The conclusions may look new to us, because we hadn't thought through the logic, but they contain no more than the information contained in the premises. They are just cast in new form, a form that may seem to give us new insight and suggest new applications, but in fact no new information or truths are generated. This is especially noticeable in mathematics, for without considerable instruction in mathematics, the deductions from even a small set of premises are not at all obvious, and may take years to develop and understand.

The bottom line is that logic alone can tell us nothing new about the real world. Ditto for mathematics, as Albert Einstein observed: "Insofar as mathematics is exact, it does not apply to reality; and insofar as mathematics applies to reality, it is not exact."

So, of what use are logic and mathematics in science? Incalculable use, once we realize their strengths and limitations. In science we construct models and theories of nature. We test and use these by deriving their logical and mathematical consequences. Logic and mathematics are the cement that holds the scientific structure together, ensures its self-consistency, and helps us prevent errors of false inference. Logic and math do not, and cannot, generate new truths about nature. They only expose and reformulate the truths contained in our models, theories and laws. The conclusions aren't *absolute* truths, but 'proximate' truths, since the raw materials upon which theories are built are based upon imperfect measurements and observations of nature. But when we know how good (how precise) the measurements are, we can also predict how good are the theories and the facts deduced from them.

Scientists do not arrive at models and theories by application of logic. They arrive at them by many processes lumped under the name 'induction'. Induction cannot be reduced to a set of logical rules (though many have tried). To see patterns (sometimes subtle and hidden ones) in data and observations requires creative ability. This is the ability to think ahead and say, "What model, set of statements (laws) or theoretical construct could I devise from which these observations and data might be deduced?"

We can't find, discover, or construct scientific laws and theories by mathematics and logic alone. But we can derive testable and useful results by application of mathematics and logic to laws and theories, and if those deduced results pass experimental tests, our confidence in the validity of the theory from which they were derived is strengthened.

In this context, logic and mathematics are reliable and essential tools. Outside of this context they are instruments of error and self-delusion. Whenever you hear a politician, theologian or evangelist casting verbal arguments in the trappings of logic, you can be pretty sure that person is talking moonshine. The quotes that open this essay reflect caution in accepting such misuses of logic.

The concern of this essay is with use and misuse of logic in science, and in discussions of the 'real world' of our experience. In the processes of science, mathematics holds a special place. While mathematics, being a subset of logic (or vice versa, you may argue), says nothing about the real world, it is the modeling tool we use for our knowledge of nature, providing the logical connection between our models and our measurements and observations. Without logic/mathematics, science as we know it is inconceivable. We would have no alternative way to integrate real-world

knowledge into a unified and useful system.

Misuse of logic is rampant in all fields, even academic ones. It is often used as a crutch to justify prejudices and as a club to smite those who hold opposing views. There are people who are thoroughly Aristotelian in their thinking, and do, indeed, believe in the profundity of empty logical arguments. Others, such as politicians and evangelists use logic cynically as an instrument for persuasion of those who don't realize that "There's a mighty big difference between good, sound reasons, and reasons that sound good." (Burton Hillis).

Just what is an 'empty' argument about the 'real world' of our experience?

One kind is the argument that may have faultless logic but is based on premises that have not, or cannot, be experimentally verified. Another kind is based on premises that are not part of any well-established and accepted scientific theory.

Some arguments are empty of content because they use words with no clear and unambiguous meaning, or words that cannot be related to anything real (experimentally unverifiable).

The most seductive empty arguments build upon premises that are so emotionally appealing that we don't ask for verification, or which have appealing conclusions that blind us to the emptiness of the premises.

Mistrust of science.

Some people are profoundly disturbed by the fact that reason alone can't generate truths. When the use of mathematics and logic in science is explained to them they respond, "If mathematics and logic can't produce absolute truths, then they produce only untruths or partial truths, and are therefore worthless." This sentence is itself an example of nonsense clothed in the appearance of logic.

It must be admitted at the outset that science is not in the business of finding absolute truths. Science proceeds as if there are no absolute truths, or if there are such truths, we can never know what they are. As the pre-Socratic skeptics observed: If we were to stumble upon an absolute truth, we'd have no way to be **certain** it is an **absolute** truth. The models and theories of science are approximations to nature—never perfect. But in most cases we know rather well *how good they are*. We can state quantitatively the limits of uncertainty of numeric results, and their range of applicability. Yet there's always the possibility that we may find exceptions to one of our accepted laws, or may even find alternative theories that do a better job than older ones.

Some critics of science attack this process of science, on the grounds that it cannot produce absolute truths. Theirs is a black/white view of the scientific process. Never mind that they have not proposed any **other** process that is capable of producing anything near the power and comprehensiveness of present science. They say that "Theory X" isn't perfect therefore it is "wrong".

The results and predictions of a theory, being well tested, will not crumble if the theory is someday modified, drastically changed, or even replaced with another theory. The results or predictions of a theory are not all suddenly rendered "wrong" when a theory is modified or replaced. These results and predictions may be improved in precision or scope. Sometimes the predictions of a new theory have greater scope than the old one, predicting things the old one didn't (and things that we never had observed or tested before). Very often a new theory is sought because the old one, while its predictions were mostly correct, predicted a few things that just weren't confirmed by good experiments. We'll need to say more about this later.

The fact that science claims no absolute truths is seized upon by people who hold strong religious beliefs and who dislike those conclusions of science that run counter to their emotional convictions. To them, if a thing is not *absolutely and finally* true, it is false, and therefore the methods used to formulate it must be flawed.

The futility of searching for absolutes.

Though the philosophers of ancient Greece developed formal logic, and got a good start toward mathematics, they

realized the limitations of logic and the futility of seeking absolutes. Here are a few comments about this dilemma.

Only one thing is certain—that is, nothing is certain. If this statement is true, it is also false.

Ancient paradox

The gods did not reveal from the beginning
All things to us; but in the course of time
Through seeking, men found that which is better.
But as for certain truth, no man has known it,
Nor will he know it; neither of the gods,
Nor yet of all the things of which I speak.
And even if by chance he were to utter
The final truth, he would himself not know it;
For all is but a woven web of guesses.
Xenophanes (c. 570-c. 480 BCE) Greek philosopher.

We know nothing in reality; for truth lies in an abyss.

Democritus, (c. 420 BCE) Greek philosopher.

None of us knows anything, not even whether we know or do not know, nor do we know whether not knowing and knowing exist, nor in general whether there is anything or not.

Metrodorus of Chios (c. 4th century BCE) Greek philosopher

This only is certain, that there is nothing certain; and nothing more miserable and yet more arrogant than man.

Pliny ("The Elder") (23-79) Roman naturalist. (Gaius Plinius Secundus).

All we know of the truth is that the absolute truth, such as it is, is beyond our reach.

*Nicholas of Cusa (1401-64) German cardinal, mathematician, philosopher. **De Docta Ignorantia** (Learned Ignorance)*

These folks who made these skeptical comments are not saying that "We can't know anything, so why bother?" They are saying that we can't "know" in the absolute sense, that we have no way to know if there are any absolute truths, and we wouldn't be able to prove the absoluteness of an absolute truth if we accidentally stumbled on one. Today we express it differently: "Science **describes** nature, it does not **explain**." Science attempts to answer "how" questions, but not "why" questions.

Science has progressed by rejecting much of its past history, past practices and past theory. Though the sciences arose from a muddled mix of mysticism, magic and speculation, scientists eventually realized that those modes of thought were prone to error and simply not productive. So chemists reject the theories of the alchemists, astronomers reject the theory underlying astrology. Mathematicians reject the number-mysticism of the Pythagoreans. Physicists, when they bother to think about their discipline's roots, acknowledge the pre-scientific contributions of the ancient Greeks in mathematics, Democritus' view that nature is lawful, and also their attitude of seeking knowledge for its own sake. But they are embarrassed by the Greek teachings about physics, for most of these have all been consigned to the trash-heap of history.

Even those early ideas that happened to be in harmony with our present views seem based upon faulty methodology or were simply speculation. Sometimes a few of those guesses seemed surprisingly close to our modern views, at least superficially. But when examined in detail the similarity breaks down. Democritus' atomic theory, for example, was based on no hard evidence, had no historical connection with modern atomic theory, and its details bore no resemblance to what we now know about atoms. Once in a while, if you speculate wildly enough, you get lucky. Too many textbooks make a "big deal" out of such accidental similarities.

Scientific method.

So, how does science arrive at its results? Some people speak of the "scientific method" as a set of "rules" for doing science. Too often such rules are presented in schools as a "recipe" for doing science, and even have numbered steps! That's misleading. At the other extreme, someone said that scientific method is "Doing one's damndest with one's mind." I know many have said better things about it, but here's some observations on scientific method.

How Science really works.

Even casual observation shows us that nature, as perceived by our senses, has reliable regularities and patterns of behavior.

Through more precise and detailed study we found that many of these regularities can be modeled, often with mathematical models of great precision.

Sometimes these models break down when extended (extrapolated) beyond their original scope of validity. Sometimes extrapolation of a model beyond its original scope actually works. This warns us that we had better rigorously test each model for validity, and these tests should be capable of exposing any flaws in the model—flaws capable of demonstrating that the model **isn't** true.

Even when a model survives such testing, we should only grant it "provisional" acceptance, because cleverer people with more sophisticated measuring techniques may in the future expose some other deficiencies of the model.

When models are found to be incomplete or deficient, we often fix them by tweaking their details till they work well enough to agree with observations.

When rapid advances in experimental observations occur, a model may be found so seriously inadequate to accommodate the new data that we may scrap a large part of it and start over with a new model. Relativity and quantum mechanics are historical examples. These situations are often called "scientific revolutions."

When such upheavals occur, and old models are replaced with new ones, that doesn't mean the old ones were totally "wrong", nor does it mean their underlying concepts were invalid. They still work within their scope of applicability. Newton's physics wasn't suddenly wrong, nor were its predictions found unreliable or incorrect when we adopted Einstein's relativity. Relativity had greater scope than Newtonian physics, but it also rested on a different conceptual basis.

Past experience has shown that mathematical models of nature have tremendous advantages over the earlier, more appealing, models which used analogies to familiar everyday phenomena of our direct sensory experience. Mathematical models are less burdened with emotional baggage, being "pure" and abstract. Mathematics provides seemingly infinite adaptability and flexibility as a modeling structure. If a some natural phenomena can't be modeled by known mathematics, we invent new forms of mathematics to deal with them.

The history of science has been a process of finding successful descriptive models of nature. First we found the easy ones. As science progressed, scientists were forced to tackle the more subtle and difficult problems. So powerful are our models by now that we often delude ourselves into thinking that we must be very clever to have been able to figure out how nature "really" works. We may even imagine that we have achieved "understanding". But on sober reflection we realize that we have simply devised a more sophisticated and detailed description.

Whatever models or theories we use, they usually include some details or concepts that do not relate directly to observed or measurable aspects of nature. If the theory is successful we may think that these details are matched in nature, and are "real" even though they are not experimentally verifiable. Their reality is supposed to be demonstrated by the fact that the theory "works" to predict things we can verify and continue to verify. This is not necessarily so. Scientists often speak of energy, momentum, wave functions and force fields as if they were on the same status as

objects of everyday experience such as rocks, trees and water. In a practical sense (for getting answers) this may not matter. But on another level, a change of scientific model may do away with a force field as an conceptual entity, but it wouldn't do away with a forest, mountain or lake.

Science progresses through trial and error, mostly error. Every new theory or law must be skeptically and rigorously tested before acceptance. Most fail, and are swept under the rug, even before publication. Others, like the luminiferous ether, flourish for a while, then their inadequacies accumulate till they are intolerable, and are quietly abandoned when something better comes along. Such mistakes will be found out. There's always someone who will delight in exposing them. Science progresses by making mistakes, correcting the mistakes, then moving on to other matters. If we stopped making mistakes, scientific progress would stop.

What do scientists really think about 'reality'?

Scientists speak in a language that uses everyday colloquial words with specialized (and often different) meanings. When a scientist says something has been found to be 'true', what is meant isn't any form of **absolute** truth. Likewise scientists' use of 'reality' and 'belief' don't imply finality or dogmatism. But if we inquire whether a scientist believes in an underlying reality behind our sense impressions, we are compounding two tricky words into a philosophical question for which we have no way to arrive at a testable answer. I'd be inclined to dismiss the entire question as meaningless, and not waste time discussing it, or any other such questions. Yet a few scientists and philosophers disagree, and wax eloquent in writing and speaking about such questions.

The notion that we can find absolute and final truths is naive, but still appealing to many people, especially non-scientists. If there are any underlying "truths" of nature, our models are at best only close approximations to them—useful descriptions that "work" by correctly predicting nature's behavior. We are not in a position to answer the philosophical question "Are there any absolute truths?" We can't determine whether there is an underlying "reality" to be discovered. And, though our laws and models (theories) become better and better, we have no reason to expect they will ever be perfect. So we have no justification for absolute faith or belief in any of them. They may be replaced someday by something quite different in concept. At least they will be modified. But that won't make the old models "untrue". All this reservation and qualification about truth, reality, and belief, doesn't matter. It isn't relevant to doing science. We can do science quite well without 'answering' these questions, questions that may not have any answers. Science limits itself to more finite questions for which we can arrive at practical answers.

Also, we've learned that not all questions we can ask have answers that we can find. Any question that is **in principle** or **in practice** untestable, is not considered a valid scientific question. We like to think that scientists don't waste time on those, but they seem to pop up in discussion and in books quite often. (Many people think unanswerable questions are the most profound and important ones. Questions like "What is the meaning of it all," or "What jump-started the universe?" I think that scientists should set these aside for the philosophers to chew on, and get on with the business of answering more accessible questions.)

Aesthetic appeal of theories.

Many who write about science emphasize the "beauty" and aesthetic appeal of successful theories. I used to naively think that to achieve intellectually and emotionally appealing theories was a goal of science. Maybe it is, on the subconscious level, as a scientist may be more enthusiastic about developing an appealing theory than an "ugly" one. And if the appealing one "works" all the ugly alternatives are dropped and forgotten.

But there's no reason why nature's operations should be beautiful or appealing to us. There's no reason why nature's operations should even be fully comprehensible to us. It could be that when we achieve an even more successful theoretical description of nature it may turn out to be messy, difficult to understand and use, and totally devoid of emotional or aesthetic appeal. We may not be capable of devising more satisfying alternatives.

We've had a taste of this already. When quantum mechanics was being developed many physicists in the forefront of developing the theory didn't "like" it, and hoped that someday they'd find a different way to formulate the theory—one more to their liking. A couple of quotes illustrate this:

Physics is very muddled again at the moment; it is much too hard for me anyway, and I wish I were a movie comedian or something like that and had never heard anything about physics!

Wolfgang Pauli (1900-1958) Austrian Physicist in the US. (Nobel Prize, 1935). From a letter to R. Kronig, 25 May 1925.

I do not like it, and I am sorry I ever had anything to do with it.

Erwin Schrödinger (1887-1961) Austrian physicist. Nobel Prize, 1933. Speaking of quantum mechanics.

In spite of great efforts to find a more appealing theory, and ingenious attempts to show that such things as the Heisenberg uncertainty principle were "wrong", the effort to remove the ugliness of quantum mechanics has (so far) failed.

It seems almost inescapable that as physics becomes more successful and more powerful its theories become farther removed from the intuitive, simple, beautiful theories of earlier centuries. This shouldn't be surprising. As we unravel the mysteries of the universe our first successes are with those accessible to direct sensory experience—phenomena that occur in everyday life and are observable without specialized apparatus, phenomena that have simple enough behavior that we can grasp the explanation and feel we "understand" it. But now we have done all the simple stuff. So we must sweat the details of phenomena that can't be directly sensed, that can only be made to occur in the lab with expensive and sophisticated equipment, and which require us to invent new mathematics to describe what's happening. The fuel that motivates us to continue along these lines is the fact that so often it works remarkably well, resulting in both scientific and technological advances. The practical technological fall-out from science stimulates funding of further research. But inevitably the science upon which the technology of our daily lives operates becomes farther removed from everyday experience and farther from the understanding of non-scientists. Most people live in a world that they understand only in a superficial way. That has been so since the beginning of human history. Yet there was a time, in fairly recent history, that almost anyone could feel that with a bit of effort and study one could learn a lot more about science, and even have a feeling of understanding much of science, and finding it intellectually and emotionally satisfying. That is much harder to do today.

I think it was Von Neumann who said that if we ever make computers that can think, with the power of the human brain or better, we won't know how they do it. Future scientific advance may be carried out entirely by computers, predicting phenomena of nature better than any previous models and theories had. But the computers by that time will be evolving independently of us, designing and re-designing themselves, learning independently of our programmers, and finding their own algorithms for dealing with nature. These algorithms will be so complex (and probably ugly) that we won't know how they work, and won't be able to re-express them in ways we can comprehend. One bit of evidence to show how this could come about is the recent fuss over the Y2K (year-2000) problem. It's terribly difficult to reconstruct the logic of programs written years ago, for which documentation is fragmentary, and the original programmers retired or deceased. Yet this problem is a small one compared to the problem of debugging a computer program written not by a human, but by a computer that is redesigning itself as it works in order to solve problems that have frustrated the few greatest minds of humanity.

The symbiotic relation between mathematics and physics.

Students and laypersons seldom grasp the difference between mathematics and physics. Since math is the preferred modeling analogy for physics, any physics textbook is richly embellished with equations and mathematical reasoning. Yet to understand physics we must realize that math is not a science, and science is not merely mathematics.

In the early history of science, mathematics was considered a "science of measurement", and was supported because of its practical applications in land measurement, commerce, navigation, etc. But those who did math discovered that mathematics was a branch of logic, and certain important results (such as the Pythagorean theorem of right triangles) could be arrived at by purely logical means without recourse to experiment. Slowly there emerged a body of knowledge called "pure" mathematics, theorems that were derived by strictly logical means from a small set of axioms. Euclid's geometry was of this form.

Today science and mathematics are separate and independent disciplines. The physicist must learn a lot of

mathematics, but the mathematician (unless working in an applied field) need not know science. In fact, most pure mathematicians seldom interact with scientists, and have no need to. Likewise, physicists generally are capable of doing mathematics without interaction with mathematicians, and have on a number of occasions, developed new mathematics to solve particularly knotty problems. One theoretical physicist I knew spent a lot of time reading the mathematics literature, saying "Those mathematics are doing some stuff that might be really useful to us. I only wish they spoke our language." His point was that the language with which each discipline speaks of its own field has diverged to the point where special effort must be made to "cross over" into the technical literature of the other field. A similar situation exists today in philosophy, where the language of philosophy of science has become so specialized and technical that most scientists find great difficulty reading it. But as one philosopher put it, "Philosophers of science observe scientists from outside, trying to figure out what they are doing, how they are doing it, and what it all means. In this process we have no need to talk to them. It's like watching a game where you don't know the rules when you come in, but try to figure out the rules by watching what the players do. For philosophers, science is a spectator sport."

Geometers can define concepts such as "circle", "triangle", "parallel lines". Within pure mathematics, these can be "perfect". The mathematician's parallel lines are strictly and perfectly equidistant from each other, to a perfection unattainable by mundane measurement. All points of a mathematician's circle are perfectly equidistant from its center, but no one could draw such a perfect circle even with the best instruments. The angles of a mathematician's triangle add to exactly 180° . But if you drew a triangle and measured the angles, each would have a finite precision and some experimental error, so the measured angles wouldn't add to exactly 180° , except accidentally.

By pure mathematics one can prove that the ratio of a circle's circumference to its diameter (called "pi") is approximately $\pi = 3.1415927\dots$, but we can also prove that one cannot express it exactly with a finite number of decimal places. Its value is an unending decimal—an irrational number. No measurement of real circles can have such perfect precision, so the value of π cannot be determined by experiment on nature. This example illustrates that mathematics propositions cannot be proven by experiment, only by pure logic. On the other hand, no scientific law or theory can be proven by using only the methods of mathematics.

The value of π is determined in the context of the axioms of Euclidean geometry. Mathematicians have also devised other, non-Euclidean, geometries. How do we even know that our universe conforms to Euclidean geometry? Measuring mechanically drawn circles is not useful for this. But we can test the geometry of space in subtler ways, and we have determined that Euclidean geometry is at least approximately true in our own cosmic neighborhood, and also out to very great distances that astronomers have observed. For "local" measurements, space is closer to Euclidean than the precision of our best measuring instruments. If we were to do such measurements and found that the angles of a triangle consistently added to something larger or smaller than 180° we would conclude that space was curved. If that were so, the value of π would be larger or smaller than the value computed from Euclidean mathematics. We would also have to inquire about the practical physical meaning of "straight" as in the "straight" sides of a triangle, or the path of a ray of light.

Mathematics is a handy analogy that can be used to model parts of nature. The mathematics can be carried out to whatever precision is needed, or "good enough" for a particular scientific purpose. Mathematics cannot discover new scientific truths, but as we develop science through hypothesis testing, mathematics can not only test the hypotheses against measurements, but help us refine (tinker) the hypothesis to bring them in closer agreement with experiment.

Logical deduction, including mathematical logic, is the language with which we frame our theories of physics. Mathematics is capable of far greater power and precision than mere words. In fact, it is the language in which many physicists do their creative thinking. It is also the tool we use to test our theories against the final (and unforgiving) arbiter of experiment and measurement. But mathematics is not a royal road to scientific truth.

—Donald E. Simanek, October 1997, March 1999, May 2002.